

Ch5 p1

MA121

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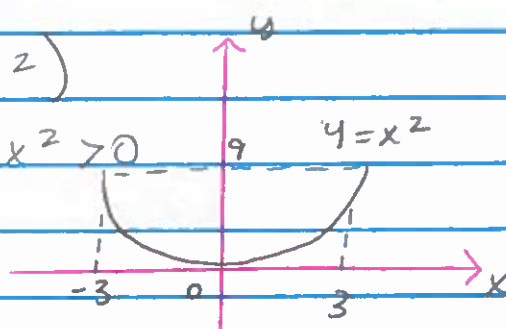
ex

$$15 P150 \quad y = \ln(9 - x^2)$$

Domain - we need $9 - x^2 > 0$

$$\Rightarrow x^2 < 9$$

$$\Rightarrow -3 < x < 3$$



Find the axes intercepts.

To find the x-intercepts, we assign $y=0$ and solve for x .

$$0 = \ln(9 - x^2)$$

$$\Rightarrow e^0 = 9 - x^2$$

$$\Rightarrow 1 = 9 - x^2$$

$$\Rightarrow x^2 = 9 - 1$$

$$\Rightarrow x = \pm\sqrt{8} = \sim 2.8284 \dots \text{ or } -2.8284$$

To find the y-intercepts, we assign $x=0$ and solve for y .

$$y = \ln(9 - 0^2) = \ln 9$$

$$\Rightarrow y = 2.1972 \dots$$

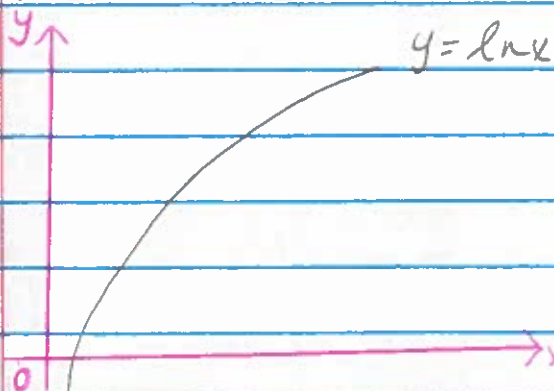
$\therefore$ x intercepts occur at $(-2.8284, 0)$ and $(2.8284, 0)$

and the y intercept is at $(0, 2.1972)$

Find asymptotes, if any.

For an $\ln$ function, if the argument approaches 0 from the right hand side, the log function approaches $-\infty$.

Recall that



So if $9 - x^2 \rightarrow 0^+$,
 $\ln(9 - x^2) \rightarrow -\infty$ or

$$y \rightarrow -\infty$$

$$\Rightarrow x^2 \rightarrow 9$$

$$\Rightarrow x \rightarrow -3 \text{ or } x \rightarrow 3$$

Un 5 p2

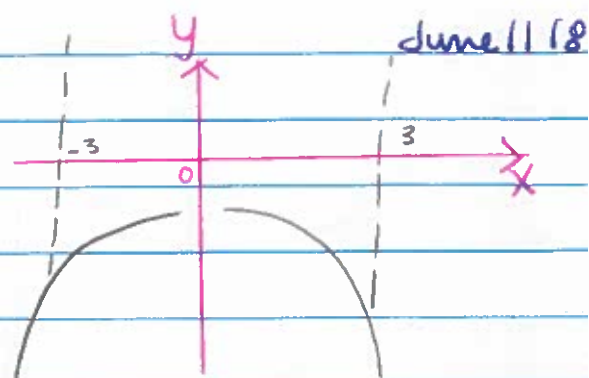
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So we conclude

lim

$$x \rightarrow -3^+ \ln(9-x^2) = -\infty$$

$$\text{and } \lim_{x \rightarrow 3^-} \ln(9-x^2) = -\infty$$

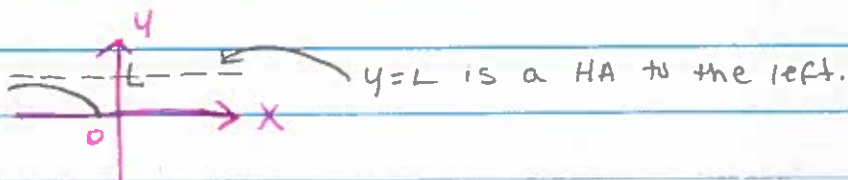


∴ we conclude that $x = -3$ and $x = 3$ are vertical asymptotes with $y = -\infty$.

Recap:

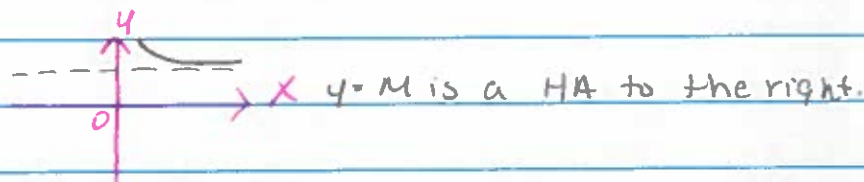
lim

$$x \rightarrow -\infty \quad y = L$$



lim

$$x \rightarrow \infty \quad y = M$$



Find intervals of increase and/or decrease and classify the relative extreme points. we need the first derivative.

$$\text{given } y = \ln(9-x^2)$$

$$i) \Rightarrow \frac{dy}{dx} = \frac{(9-x^2)'}{9-x^2} = \frac{-2x}{9-x^2}$$

To find critical points,

$$\frac{dy}{dx} = 0 \Rightarrow \frac{-2x}{9-x^2} = 0 \Rightarrow -2x = 0 \Rightarrow x = 0 \leftarrow \text{accepted as it is on the domain}$$

$$ii) \frac{dy}{dx} = \text{DNE} \Rightarrow \frac{-2x}{9-x^2} = \text{DNE} \Rightarrow 9-x^2 = 0 \Rightarrow x^2 = 9 \Rightarrow x = -3 \text{ or } x = 3. \text{ These critical points are not on the domain.}$$

∴ $y = \ln(9-x^2)$ has only one critical point i.e. $x = 0$

We now perform a sign analysis on

$$\frac{dy}{dx} = \frac{-2x}{9-x^2} \dots\dots$$

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	-3	$-3 < x < 0$	0	$0 < x < 3$	3
Sign of $-z$		negative		negative	
Sign of x		negative		positive	
Sign of $9-x^2$		positive		positive	
Sign of $\frac{dy}{dx} = \frac{-2x}{9-x^2}$		positive		negative	
orientation of $y = \ln(9-x^2)$		incline		decline	

at $x=0$, then $y = \ln(9-x^2) = \ln 9 \approx 2.1972...$

∴ relative maximum point at $(0, 2.1972)$

Find intervals of concave up, concave down, and determine inflection points, if any. We need the second derivative.

$$\text{from } \frac{dy}{dx} = \frac{-2x}{9-x^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{(9-x^2)(-2x)' - (-2x)(9-x^2)'}{(9-x^2)^2}$$

$$= \frac{(9-x^2)(-2) + (2x)(-2x)}{(9-x^2)^2}$$

$$= \frac{-18 + 2x^2 - 4x^2}{(9-x^2)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2x^2 - 18}{(9-x^2)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2(x^2 + 9)}{(9-x^2)^2}$$

Inflection points may occur when

$$i \quad \frac{dy}{dx} = 0 \Rightarrow \frac{-2(x^2 + 9)}{(9-x^2)^2} = 0 \Rightarrow -2(x^2 + 9) = 0 \Rightarrow x^2 + 9 = 0 \Rightarrow x^2 = -9$$

But $x^2 = -9$ has no solution. So $\frac{d^2y}{dx^2}$ does not yield any possible inflection points for this function.

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ii $\frac{d^2y}{dx^2} = \text{DNE} \Rightarrow \frac{-2(x^2+9)}{(9-x^2)^2} = \text{DNE} \Rightarrow (9-x^2)^2 = 0 \Rightarrow 9-x^2 = 0$
 $\Rightarrow x^2 = 9 \Rightarrow x = -3$ or $x = 3$. We reject these as possible inflection points as they are not on the domain. $\therefore$ This function $y = \ln(9-x^2)$ has no inflection points.

-3 3

sign of -2

negative

sign of x^2+9

positive

sign of $(9-x^2)^2$

positive

sign of $\frac{d^2y}{dx^2} = \frac{-2(x^2+9)}{(9-x^2)^2}$

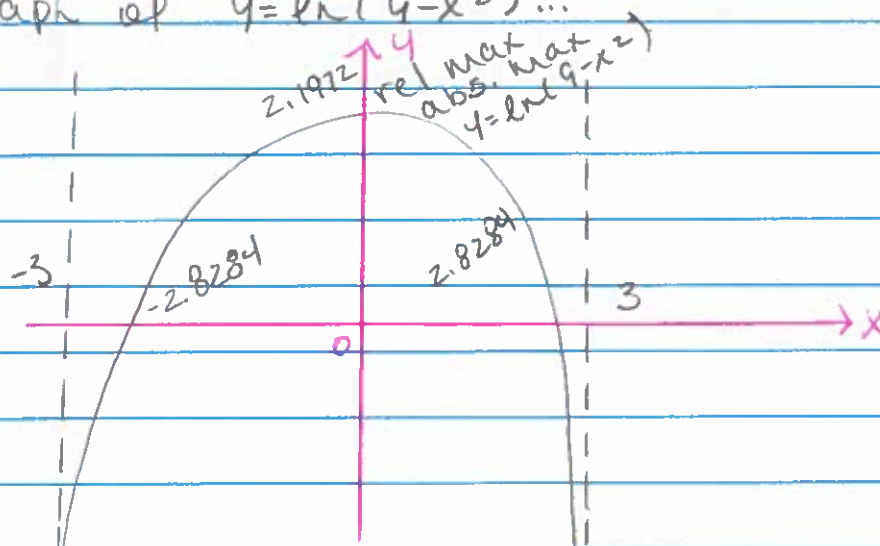
negative

concavity of $y = \ln(9-x^2)$



rel. max. at $(0, 2.1972)$

Graph of $y = \ln(9-x^2)$...



S.5

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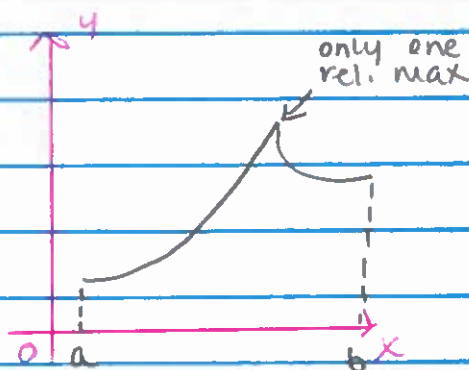
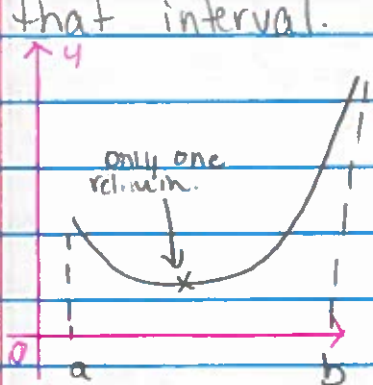
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MID 3 will cover select topics from chapter 4 up to (including) 5.4

FINAL EXAM = Entire text. NO FORMULA SHEETS.

Applications of Extreme Values / Unique Extreme Theorem

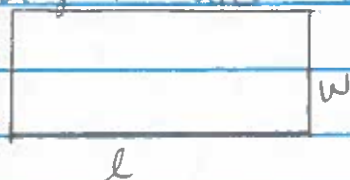
If a continuous function has exactly one relative minimum (or maximum) on an interval, then that value is also an absolute minimum (or maximum) on that interval.



* guidelines on solving extreme values problems on p. 157.

ex 1 p 162

Peach's garden is to be planted on a rectangular lot. The lot must have a perimeter of 750 ft. What are the dimensions of the rectangle that encloses the largest area on the lot?



let l = length and w = width.

We know that perimeter, $P =$ and given $P = 750$,

$$P = l + l + w + w$$

$$750 = 2(l + w)$$

$$\Rightarrow P = 2l + 2w$$

$$\Rightarrow l + w = \frac{750}{2}$$

$$\Rightarrow P = 2(l + w)$$

$$\Rightarrow l + w = 375$$

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we want the area to be maximized.

$A = lw$; It may appear that A is in terms of two variables. However, we can rewrite

$l + w = 375$ as l in terms of w or vice versa, and substitute back into $A = lw$ to obtain A in terms of only one variable.

$$l + w = 375$$

$$\Rightarrow l = 375 - w \quad \leftarrow \text{Substitute into } A = lw$$

$$\Rightarrow A = (375 - w)(w)$$

$$\Rightarrow A = 375w - w^2$$

Now we find the absolute value for A and what the values of l and w are that yields this absolute value of A . Domain = $0 \leq w \leq 375$.

Now we need the first derivative.

$$\frac{dA}{dw} = 375 - 2w \quad \text{critical points occur when}$$

- i $\frac{dA}{dw} = 0 \Rightarrow 375 - 2w = 0 \Rightarrow w = \frac{375}{2} = 187.5 \text{ ft.} \leftarrow \text{accepted point}$
- ii $\frac{dA}{dw} = \text{DNE}$, but $\frac{dA}{dw} = 375 - 2w$ is a polynomial, so it

exists everywhere on the domain. This criteria does not yield any critical points. $\therefore A = 375w - w^2$ has only one critical point i.e $x = 187.5 \text{ ft.}$ Now, to classify this critical point, we can use the second derivative test.

$$\text{From } \frac{dA}{dw} = 375 - 2w \Rightarrow \frac{d^2A}{dw^2} = -2. \quad \text{we have, at } w = 187.5$$

$$\frac{dA}{dw} = 0 \quad \text{and} \quad \frac{d^2A}{dw^2} = \text{negative, so this critical point, } w = 187.5$$

is a relative maximum point. we know that

$A = 375w - w^2$ is continuous on its domain and

$w = 187.5$ is the only relative maximum point

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on the domain. So according to the unique Extremum Theorem, this relative max point is also a relative minimum point.

In conclusion,

$W = 187.5 \text{ ft}$ ← this area is maximised when

$$W = 187.5 \text{ ft} \quad l = 375 - W$$

$$= 375 - 187.5 = 187.5$$

The maximum area that Peach can attain for her garden with a perimeter of 750 ft occurs when $l = 187.5 \text{ ft}$ and $W = 187.5 \text{ ft}$.